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DISPERSION PHENOMENON IN EXTRUDING A MATERIAL FROM A VESSEL THROUGH A NARROW HOLE

M. K. Zhekamukhov and L. Z. Shukhova

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A hydrodynamic model of the process of dispersion of solids extruded by a piston from a vessel through a narrow hole is considered. The results obtained on the basis of this model are applied to evaluate the drilling depth at which the borehole walls begin to lose their stability.

As is well known, one way of producing wire is by squeezing out a solid material from a vessel under high pressure [1]. If this process is conducted directly by a piston, then because of the friction near the vessel walls the pressure will not be transferred to the hole through which extrusion is carried out. To avoid this difficulty, the vessel is filled with a liquid and the extrusion is carried out by means of hydrostatic pressure. The extrusion occurs here only on attaining the yield stress of the material by the hydrostatic pressure, which for softer materials is much lower than those pressures which can be sustained by steel vessels. In [1], it is noted that this method allows one to extrude even steel from steel vessels if the hole is sufficiently small. Moreover, the effect of interest has been revealed: with decrease in the hole and increase in the pressure, the metal ceases to smoothly emerge from the hole and begins to be ejected in individual pieces. Further increase in the pressure leads to a strong heating caused by the friction. And if the material is heated to its annealing temperature, the advantages of the extrusion process as a method of producing wire disappear.

The phenomenon of dispersion of solid bodies in sudden relieving of uniform-compression stresses has also been considered in [2]. In this work, it is suggested to use this phenomenon as an economical method of preparing highly dispersed powders; threshold pressures at which the process of dispersion of the material in its fast unloading begins are also presented in this work for a wide range of materials.

As far as we know, the process of dispersion of a solid body in its extrusion from a vessel has not yet been investigated theoretically.

In the present work, we suggest a theoretical model of this phenomenon.

Figure 1 (taken from [1]) gives a diagram of the installation for extruding a solid material from a vessel. The extruded material A is under the hydrostatic pressure p of liquid B. In what follows, we will assume the pressure to be constant.

As is known from the mechanics of deformable bodies, at pressures exceeding the dynamic strength of solid bodies, the latter change to a yield state and look like fluids. The motion of the substance at such high pressures is described by ordinary hydrodynamic equations. In particular, in extruding the substance from a container through a hole at a constant velocity, the Bernoulli equation is fulfilled:

$$\frac{u^2}{2} + \int \frac{dp}{\rho} = \text{const} . \quad (1)$$

To expand the integral in Eq. (1), it is necessary to prescribe the equation of state of the substance. It is a known fact [3] that in the course of compression and unloading of substances up to several hundred thousand atmospheres, the change in the entropy is insignificant; therefore, the pressure depends only on the density or the volume. In the range of pressures not exceeding the ultimate strength of a solid body, the unloading in the body is described by formulas of elasticity theory.

Kabardino-Balkar State University, Nal'chik, Russia; email: jekamuhova@mail.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 3, pp. 96–99, May–June, 2002. Original article submitted April 4, 2001; revision submitted October 8, 2001.

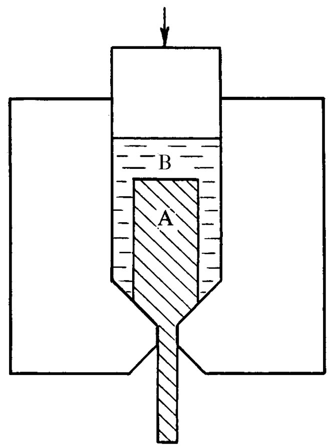

Fig. 1. Diagram of the installation for extruding the material under hydrostatic pressure: A, material to be extruded; B, liquid.

There are different empirical and semiempirical equations of state of solid bodies. In the range of pressures to several hundred thousand atmospheres the most simple and convenient is the equation of the form

$$p = A\left[\left(\frac{\rho}{\rho_0}\right)^n - 1\right], \tag{2}$$

in which the coefficient A and the exponent n can be considered to be constant and related by the relation

$$An = \rho_0 c_0 , \tag{3}$$

where $c_0 = (K/\rho_0)^{1/2}$ at $p = 0$. From the data of [4], the coefficient A is equal to $4.5 \cdot 10^8$ for iron, $2.5 \cdot 10^8$ for copper, and $2.03 \cdot 10^8$ kN/m^2 for duralumin; for metals the exponent n can be taken to be equal to four.

Neglecting the velocity of the substance in the vessel, from equalities (1) and (2) we will have

$$u^2 = \frac{2c_0^2}{n-1}\left[\left(1 + \frac{p_s}{A}\right)^{(n-1)/n} - \left(1 + \frac{p}{A}\right)^{(n-1)/n}\right]. \tag{4}$$

Hence for the maximum velocity u_{max} of the jet the following formula is obtained:

$$u_{max} = c_0\left[\frac{2}{n-1}\right]^{1/2}\left[\left(1 + \frac{p_s}{A}\right)^{(n-1)/n} - 1\right]^{1/2}; \tag{5}$$

for $p_s << A$ $u_{max} = (2p_s/\rho_0)^{1/2}$.

On leaving the hole, the substance is extended. At the site of the jet cross section where the tensile stress reaches the ultimate tensile strength of the body, the latter is ruptured and in the case of a rather intense outflow of the substance from the hole its spraying occurs.

Usually, because of the microscopic inhomogeneity of the material the zone of tensile stresses is smeared, while the rupture surface is rough. However, in what follows this roughness will be neglected and it will be assumed that the process of dispersion of the material occurs along a fixed plane cross section at the site of the flow where the tensile stress becomes equal to the dynamic ultimate tensile strength of the material. We call this cross section the "dispersion front" of the material and will mark the flow parameters in this cross section by an asterisk.

Upon change in the direction of deformation, the material in the extension zone acquires the capacity for elastic deformation again. It can be assumed that this capacity is retained up to the onset of rupture. Thus, as the pressure changes from $p = 0$ to $p = -\sigma^*$ in the extension zone of the material, the equation of state in the form of the Hooke law is suitable. On the other hand, for $\sigma^*/A << 1$ the equation of state (2) changes to the Hooke law. Hence it follows that over the entire range of variation of the pressure from $p = p_s$ to $p = -\sigma^*$ we can use Eq. (2).

Taking into consideration the fact that at $p = 0$ the velocity is $u = u_{max}$, from equality (1) we obtain

$$u^2 = u_{max}^2 - \frac{2\,|p|}{\rho}\,, \quad -\sigma^* \le p \le 0\,. \tag{6}$$

On the "dispersion front," we have the following relation:

$$u^* = \left(u_{max}^2 - \frac{2\sigma^*}{\rho^*}\right)^{1/2}. \tag{7}$$

Hence for $p_s << A$ we will have

$$u^* \approx \left[\frac{2\,(p_s - \sigma^*)}{\rho_0}\right]^{1/2}. \tag{7'}$$

The condition of rupture of the material is reduced to the inequality

$$\rho^* u^{*2} \ge \sigma^*\,. \tag{8}$$

Using equalities (5) and (7) as well as relation (3) and the equation of state (2), we can rewrite condition (8) in the form

$$\frac{2K}{n-1}\left[\left(1 + \frac{p_s}{A}\right)^{(n-1)/n} - 1\right]\left(1 - \frac{\sigma^*}{A}\right)^{1/n} \ge 3\sigma^*\,.$$

Virtually for all the solids the ratio $\sigma^*/A << 1$ holds; therefore, we represent the last condition as

$$\left(1 + \frac{p_s}{A}\right)^{(n-1)/n} - 1 \ge 3\,\frac{n-1}{p}\,\frac{\sigma^*}{K}\,. \tag{9}$$

Whence, in an acoustic approximation, when $p_s << A$, we obtain

$$p_s \ge p_t = \frac{3}{2}\,\sigma^*\,, \tag{10}$$

where p_t is the threshold value of the pressure at which the material begins to rupture. Thus, for example, according to [5], for steels of different grades the values of σ^* are within the limits from $5\cdot10^6$ to $15\cdot10^6$ kN/m^2, consequently, $p_t \approx (8–23)\cdot10^6$ kN/m^2; for aluminum we have $\sigma^* = (2–4)\cdot10^6$ kN/m^2, and the corresponding value of p_t is equal to $(3–6)\cdot10^6$ kN/m^2. These results are in satisfactory agreement with the experimental data [2].

It should be noted that formula (10) can be used to solve the inverse problem, i.e., to determine the dynamic tensile strength of the material. To do this, it is sufficient to find the threshold value of the pressure p_t for a given material and then to determine the quantity σ^* from formula (10).

The dispersion of the material begins at pressures p_s exceeding the threshold pressures. In this case, on the "dispersion front" the laws of conservation of mass, momentum, and energy must be obeyed. These laws are written in just the same way as for shock waves:

$$\rho^* u^* = \rho_1 u_1\,, \tag{11}$$

$$-\sigma^* + \rho^* u^* = \rho_1 u_1 , \tag{12}$$

$$\frac{\sigma^*}{\rho^*} + \frac{u^{*2}}{2} = \frac{W}{\rho_1} + \frac{u_1^2}{2} . \tag{13}$$

Here $\rho_1 = \rho_0$. The quantity W can be represented in the form $W = d^2 N\gamma$.

From equalities (11) and (12) it follows that

$$u_1 = u^* - \frac{\sigma^*}{\rho^* u^*} = \left(\frac{2}{\rho_0}\right)^{1/2} \left(p_s - \frac{3}{2}\sigma^*\right)(p_s - \sigma^*)^{-1/2} . \tag{14}$$

Hence it is evident that $u_1 = 0$ for $p_s = 3\sigma^*/2$, while the difference of u_1 from u^* is insignificant for $p_s >> \sigma^*$, i.e., at the pressures $p_s >> \sigma^*$ the energy expended on dispersing the body is low compared to the potential energy stored in the body.

Equality (13) can be represented in the form

$$\frac{u_{max}^2}{2} = \frac{W}{\rho_0} + \frac{u_1^2}{2} .$$

The meaning of this equality is apparent: the potential energy of the compressed substance changes completely to the kinetic energy of the substance per unit volume $u_{max}^2/2$. The latter in the process of unloading of the material changes to the kinetic energy of the particles and is also expended on dispersing the material.

We note that relation (10) is only the condition of rupture of the material, and for the material to be dispersed a more rigid requirement must be fulfilled.

As is well known, macroscopic bodies contain many fine cracks by which the rupture of a body occurs. Because of their existence the technical strength of solids is two to three orders of magnitude lower than the theoretical strength, which is approximately equal to the elastic modulus E.

According to Frenkel' [6], the minimum stress at which the crack begins to elongate (and the body to rupture) is determined by the formula $p_{min} = (2\gamma E/d')^{1/2}$. The size of the particles formed in rupture of a solid body are of the same order of magnitude as the size of the fine cracks contained in this body, i.e., $d' \approx d$. Thus, for the particles of size d to be formed it is necessary that the pressure in the vessel satisfy the condition

$$p_s \geq 1.5 p_{min}(d) \approx 1.5 \left(\frac{2\gamma E}{d}\right)^{1/2} . \tag{15}$$

Hence it is evident that as the pressure preceding the unloading of the body increases, the size of the particles formed during the dispersion of the substance decreases following the law $d \sim 1/p_s^2$.

Relation (15) closes the system of equations (11)–(13) and is used as a certain "equation of state" that relates the pressure p to the size of the particles formed in dispersion of bodies.

To orient ourselves in the values of the parameters determining the process of dispersion of solid bodies, we consider the data for iron: $\sigma^* = 4\cdot 10^8$ N/m^2, $c_0 = 4.63\cdot 10^3$ m/sec, $E = 3.4\cdot 10^{11}$ N/m^2, $\gamma = 1$ J/m^2, and $\rho_0 = 7.5\cdot 10^3$ kg/m^3. We set $p_s = 4\sigma^* = 10^9/6$ N/m^2. Then we will have $u_{max} = 600$ m/sec, $u_1 \approx 500$ m/sec, and $d \sim 0.25$ μm.

The results obtained above can be used in geophysics.

As is well known [7], the depth of the largest boreholes on the continents is 8 to 9 km, while the Kola superdeep borehole is 13 km in depth. The mining pressure from the overlying rocks at a depth of H is determined by the formula $p_1 = \bar{\rho} g H$, where $\bar{\rho}$ is the mean density of the overlying rocks. On the average, $\bar{\rho} g = 3.5\cdot 10^4$ N/m^3. Thus, at a depth of 10 km the pressure p_1 is $3.5\cdot 10^8$ N/m^2. Such high pressures are already sufficient for the spontaneous process of dispersion of rocks to develop, i.e., for the borehole walls to lose their stability, at faces during superdeep drillings.

The process of extrusion of a substance from a face into a borehole is quite similar to the above-considered process of extrusion of a substance from a vessel: the granite layer of the continental earth crust of thickness H plays the role of vessel walls, while instead of p_s we have the mining pressure $p_1 = \bar{\rho} g H$.

The depth H^*, at which the stability of the borehole walls is lost, can be determined by the equality $\bar{\rho} g H^* = 3\sigma^*/2$, i.e., $H^* = 3\sigma^*/(2\bar{\rho} g)$, where σ^* is the dynamic tensile strength of the rocks.

On the other hand, in [8], for the initial depth of cavern formation the formula $H^* = 2\sigma_c/(\bar{\rho} g)$ has been obtained by the methods of elasticity theory; in this formula, σ_c is the empirical constant selected in such a way that in the investigated range of stresses the theoretical value of the quantity H^* coincides with the value determined experimentally.

If we set $\sigma_c = 3\sigma^*/4$, both results for H^*, which are found by quite different methods, coincide.

Below the depth H^*, the rock ground at the face loses its stability, changes to the yield state, and ceases to exhibit resistance to shear deformation. The boring tool at the face will not experience resistance. The depth H^* is determined as the limiting depth of the borehole and depends mainly on the dynamic tensile strength of the rock.

NOTATION

u, velocity; p, pressure; ρ, density of the substance; ρ_0, density of the substance at $p = 0$; A, empirical constant; c_0, velocity of the plastic waves at $p = 0$; K, modulus of uniform compression; p_s, initial pressure in the vessel; n, adiabatic exponent; u_{max}, maximum velocity of outflow of the substance from the hole; σ^*, ultimate tensile strength of the material; u^*, velocity on the "dispersion front"; ρ^*, density on the "dispersion front"; p_t, threshold pressure in the vessel at which the material is dispersed; ρ_1, density of the particles formed in dispersion of the material; u_1, mean velocity of the particles formed in dispersion of the material; W, energy required for dispersion of a unit volume of the substance; N, number of particles in the unit volume; γ, surface energy of the dispersed material; d, mean size of the particles; E, Young modulus; d', size of the cracks in the material; p_{min}, minimum stress at which the crack in the material elongates; p_1, mining pressure; g, acceleration of gravity; H, depth of occurrence of the rock; H^*, critical depth of the boreholes at which the borehole walls lose their stability; σ_c^*, empirical constant having the dimension of stress.

REFERENCES

1. P. W. Bridgman, *Studies in Large Plastic Flow and Fracture* [Russian translation], Moscow (1955).
2. V. T. Fedorov and Kh. B. Khokonov, *Dokl. Akad. Nauk SSSR*, **300**, No. 5, 1126–1128 (1988).
3. Yu. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Flows* [in Russian], Moscow (1966).
4. F. A. Baum, K. P. Stanyukovich, and B. I. Shekhter, *Physics of Explosion* [in Russian], Moscow (1959).
5. L. V. Al'tshuler, *Usp. Fiz. Nauk*, **85**, Issue 2, 197–258 (1965).
6. Ya. I. Frenkel', *Introduction to the Theory of Metals* [in Russian], Moscow (1958).
7. S. A. Chechkin, *Principles of Geophysics* [in Russian], Leningrad (1990).
8. G. A. Cherepanov, *Prikl. Mekh. Tekh. Fiz.*, **94**, No. 2, 146–158 (1982).